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A Practical Model for Computing Subsurface BRDF of Homogeneous Materials with A Thin Layer of Paint

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Abstract This paper presents a practical model for computing the subsurface BRDF of materials composed of a homogeneous semi-infinite layer and a thin painted layer on the top. Solution of light propagation inside a scattering/absorbing medium is expensive. We propose a fast and exact subsurface BRDF computation model using the Ambartsumian's integral equation for the bottom layer and using invariant imbedding method to modify the subsurface BRDF when a thin layer of material is painted on top. By using this model, we avoid the expensive light transport computation inside the material and compute the subsurface BRDF of the material directly. We validate our model against the solutions from a standard discrete ordinate (DOM) based light transport solver and demonstrate our model by simulating a variety of materials.

Keywords BRDF · light transport · realistic image synthesis · invariant imbedding · Ambartsumian's integral equation

1 Introduction

Accurately modeling the light interaction with the material is important and challenging for realistic image synthesis. Although many physics processes are involved in this interaction, the reflectance profile is of particular importance for its prominent visual effects. Reflection, usually described as a bidirectional reflectance distribution function (BRDF) [13], can be broadly separated into two parts: surface and subsurface BRDF as described by Hanrahan and Krueger [8]. Though for computational convenience the subsurface BRDF along with the diffuse part of the surface BRDF is often approximated as directionally independent Lambertian term, for most real world materials the subsurface BRDF is directionally dependent [8], which means subsurface BRDF may not be a constant for all outgoing directions. Kubelka-Munk method [11] is an often used method for computing directionally independent subsurface BRDF. Most of the recent works on subsurface BRDF have been based on the diffusion approximation theory [7], which also do not take into account the directional property. Although Blinn [2] used single scattering approximation to address the directional dependency problem, a physically correct subsurface BRDF model must take into account multiple subsurface scattering accurately.

Adding and doubling method [5] and Discrete Ordinate Methods (DOM) [17], two standard ground truth methods for light transport computation, can in principle be used to compute subsurface BRDFs. Both of these accurate methods compute radiance field for the whole volume, and hence are expensive. As BRDF ultimately relates only the outgoing radiation field at the boundary to the incident radiation, radiation field computed for the bulk of the material does not provide any useful information and hence the effort involved in computing them can be considered as wasteful. So for efficient BRDF computation any method that allows us to compute the radiance field only at the boundary would be a preferable choice. The search for such a method led us to the Ambartsumian's method [4, 12, 15]. Our method for computing subsurface BRDF of the thick layer material(used as bottom layer in this paper), falls into this category.

In this paper, we present a practical model for accurately computing the directionally dependent subsur-

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face BRDF for homogeneous bulk materials with a thin painted layer whose optical properties may be different. Our subsurface scattering analysis for the bottom layer is based on the theory introduced by Ambartsumian [1]. For the painted layer, we make use of the *invariant imbedding equation* [9], introduced by Pharr and Hanrahan [14], to derive a formula for computing its subsurface BRDF. Finally we give the complete BRDF solution for this two-layered materials by adding the surface BRDF.

2 Theory

Light interaction by subsurface scattering can be modeled using radiative transfer theory [3]. If the material is homogeneous and optically thick(no transmittance), an analytical formula can be derived from the plane-parallel Radiative Transfer Equation (PRTE) [3]:

$$\begin{aligned} R^m(\eta_o,\eta_i) &= \frac{\alpha}{4(\eta_o+\eta_i)} p^m(-\eta_o,\eta_i) \\ &+ \frac{\alpha\eta_i}{2(\eta_o+\eta_i)} \int_0^1 p^m(\eta_o,\eta') R^m(\eta',\eta_i)\mathrm{d}\eta' \\ &+ \frac{\alpha\eta_o}{2(\eta_o+\eta_i)} \int_0^1 p^m(\eta',\eta_i) R^m(\eta_o,\eta')\mathrm{d}\eta' \\ &+ \frac{\alpha\eta_o\eta_i}{(\eta_o+\eta_i)} \int_0^1 R^m(\eta_o,\eta')\mathrm{d}\eta' \int_0^1 p^m(\eta',-\eta'') R^m(\eta'',\eta_i)\mathrm{d}\eta''. \end{aligned} \tag{1}$$

Here the R^m and p^m represent the m^{th} order Fourier expansion coefficients of the reflectance function and the phase function, respectively. α is the single scattering albedo. η_o stands for the cosine value of the scattering zenith angle, and η_i for the cosine value of the incident light beam. This equation is termed *Ambartsumian's integral equation*, which relates the Fourier coefficients of the subsurface BRDF by an integral equation with single scattering albedo and the Fourier coefficients of the phase function as the known terms. So these integral equations can be numerically solved to compute the subsurface BRDF coefficients and in turn the subsurface BRDF.

In order to accurately model how subsurface BRDF changes according to a painted thin layer of material at the surface, the *invariant imbedding equation* is used [9]. The subsurface BRDF of the painted layer can be computed as follows:

$$\begin{aligned} R^m_{\text{modified}}(\eta_o,\eta_i) &= \left(1-\tfrac{\Delta\tau}{\eta_o}\right) R^m(\eta_o,\eta_i) \left(1-\tfrac{\Delta\tau}{\eta_i}\right) \\ &+ \tfrac{\alpha\Delta\tau}{4\eta_o\eta_i} p^m(-\eta_o,\eta_i) \\ &+ \tfrac{\alpha\Delta\tau}{2\eta_i} \textstyle\int_0^1 p^m(\eta',\eta_i) R^m(\eta_o,\eta')\mathrm{d}\eta' \\ &+ \tfrac{\alpha\Delta\tau}{2\eta_o} \textstyle\int_0^1 p^m(\eta_o,\eta') R^m(\eta',\eta_i)\mathrm{d}\eta' \\ &+ \alpha\Delta\tau \textstyle\int_0^1 R^m(\eta_o,\eta')\mathrm{d}\eta' \int_0^1 p^m(-\eta',\eta'') R^m(\eta'',\eta_i)\mathrm{d}\eta'', \end{aligned} \tag{2}$$

where $\Delta\tau$ is the optical thickness of the painted layer.

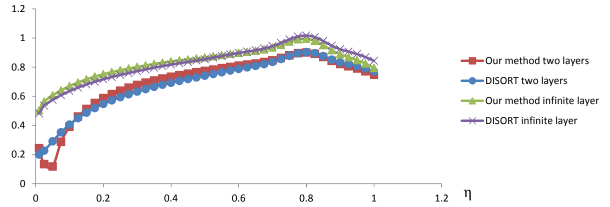

Fig. 1 Results compared with DISORT: the plotted data shows the subsurface BRDF value at 180^o azimuth angle for blue wavelength. The material used to generated the subsurface BRDF is TiO_2 as the semi-infinite layer and AlGaAs7 as the top layer with 0.1 optical depth.

3 Numerical computation

Numerical method for computing the Ambartsumian's integral equation and the invariant imbedding equation is discussed in this section.

3.1 Solving R^m and R^m_{modified}

For numerical computation, we convert equation 1 into a system of $n \times n$ algebraic equations:

$$\begin{aligned} &(\eta_p+\eta_q) R^m(\eta_p,\eta_q) = \tfrac{\alpha}{4} p^m(-\eta_p,\eta_q) \\ &+ \tfrac{\alpha\eta_q}{2} \textstyle\sum_{s=1}^n w_s p^m(\eta_p,\eta_s) R^m(\eta_s,\eta_q) \\ &+ \tfrac{\alpha\eta_p}{2} \textstyle\sum_{s=1}^n w_s p^m(\eta_s,\eta_q) R^m(\eta_p,\eta_s) \\ &\quad + \alpha\eta_p\eta_q \textstyle\sum_{s=1}^n w_s R^m(\eta_p,\eta_s) \times \\ &\times \textstyle\sum_{s'=1}^n w_{s'} p^m(\eta_s,-\eta_{s'}) R^m(\eta_{s'},\eta_q). \end{aligned} \tag{3}$$

Before iteratively solving this equation, we first compute $p^m(-\eta_p,\eta_q)$ and $p^m(\eta_p,\eta_q)$ for each pair of (p,q), where η_p and η_q are the selected n quadrature nodes in the interval $[0,1]$. For accurate and fast integration, a modified Gaussian quadrature scheme is chosen, which is originally proposed by Mishchenko et al. [12]. The quadrature nodes and weights are described as follows:

$$\eta_n = cos(\frac{\pi}{4}G_n + \frac{\pi}{4}), \qquad w_n = \frac{\pi}{4}W_n sin(\frac{\pi}{4}G_n + \frac{\pi}{4}), \tag{4}$$

where G_n and W_n are the Gaussian quadrature nodes and weights respectively.

In order to start the iteration, the initial value for $R^m(\eta_p,\eta_q)$ is set using single scattering approximation.

We apply the same quadrature technique to equation 2 to compute the modified subsurface BRDF due to the added thin layer.

3.2 Generating BRDF

The subsurface BRDF for semi-infinite base layer and two layer material is reconstructed from R^m and R^m_{modified},

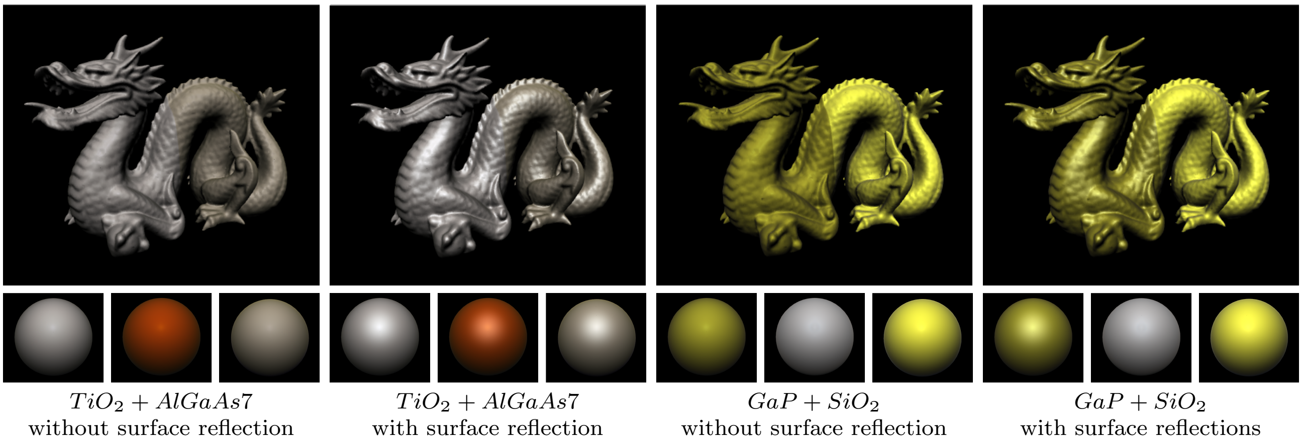

Fig. 2 A dragon model whose left half is rendered using BRDF of the semi-infinite base and right half with a thin layer painted on the top. The first and third images on the top row are rendered with subsurface BRDF, the second and fourth images are rendered using the complete BRDF (surface+subsurface). Associated with each dragon image are the images of three spheres that are rendered using the BRDF for the base material, the painted material and the two-layer material, respectively.

Table 1 Averaged diffuse reflectance factor

Material	R	G	B
GaP	0.68224	0.66708	0.64471
SiO_2	0.90465	0.90404	0.90343
TiO_2	0.72505	0.71400	0.70362
$AlGaAs7$	0.67603	0.65280	0.63531

respectively. Moreover, we model surface BRDF using Torrance-Sparrow BRDF [19].

Any light that is not reflected at the surface is assumed to transmit into the material and undergoes subsurface scattering. So the subsurface BRDF should be modulated by *diffuse reflectance factor* [6]: $(1-\rho_{dt}(\omega_i))$. Thus the complete BRDF becomes the combination of the surface BRDF R_s and the subsurface BRDF R_d:

$$R(\omega_i, \omega_o) = R_s(\omega_i, \omega_o) + (1 - \rho_{dt}(\omega_i))R_d(\omega_i, \omega_o). \quad (5)$$

4 Results

We assume that the material is composed of spherical particles, hence the optical properties, the single scattering albedo and the phase function of the materials can be computed using Mie theory [10]. For the Mie theory computation, we choose the power law distribution [9] for the particle size distribution and the refractive index of the material is taken from SOPRA optical database [16]. The subsurface BRDF computation takes few seconds. We validated our results with results from DISORT [18]. The comparison is shown in Figure 1. Note that as η approaches to zero, the terms on the right hand side of equation 2 with η in the denominator approach to infinity and so does R^m_{modified}. To handle this situation, we use single scattering approximation to compute R^m_{modified} at small η values. This results in a small mismatch with DISORT's output at those η values.

Figure 2 displays rendering results using our subsurface BRDF model for two different cases: One with Titanium Dioxide (TiO_2) as the base layer and Aluminum Gallium Arsenide (70% Al)($AlGaAs7$) as a thin painted layer at the top. The other with Gallium Phosphide (GaP) as the base and Silicon Dioxide(SiO_2) at the top. The effective radius and effective variance for power law particle size distribution is set to $50\mu m$ and 0.2, respectively. Also, the roughness parameter for the Beckmann microfacet distribution in the surface BRDF computation is set to 0.3, and the optical depth of the top layer is set to 0.1. Since the diffuse reflectance factor of SiO_2 is high(see Table 1), most of the light is transmitted into the material, the subsurface BRDF is predominant in this case, so the difference is less between BRDF with and without surface BRDF.

Figure 3 shows the rendering of CGI logo using BRDF from different base and thin layer materials. The background is composed of base semi-infinite material and the text in the logo is modeled as a thin layer on the top of the base. Different roughness values are used for the materials used in the text in the logo. The roughness for the right side of the text is set to 0.1 and 0.3 for left side. We use Henyey-Greenstein phase function for the top layer. The optical properties of the top layer is shown in Table 2.

5 Conclusion and future work

We have presented a model for rendering two-layered materials with a semi-infinite homogeneous layer at the

Table 2 Optical properties(single scattering albedo, Henyey-Greenstein phase function's asymmetry parameter and refractive index) of the painted materials used for CGI logo

Optical Properties	Logo 1	Logo 2	Logo 3	Logo 4
albedo	$(0.10, 0.90, 0.93)$	$(0.70, 0.90, 0.33)$	$(0.70, 0.90, 0.33)$	$(0.14, 0.90, 0.93)$
asymmetry parameter	$(0.54, 0.46, 0.23)$	$(0.44, 0.36, 0.50)$	$(0.45, 0.38, 0.50)$	$(0.54, 0.49, 0.23)$
refractive index	$(1.66, 3.16, 3.06)$	$(3.25, 2.56, 2.06)$	$(3.26, 2.56, 2.06)$	$(1.66, 3.16, 3.06)$

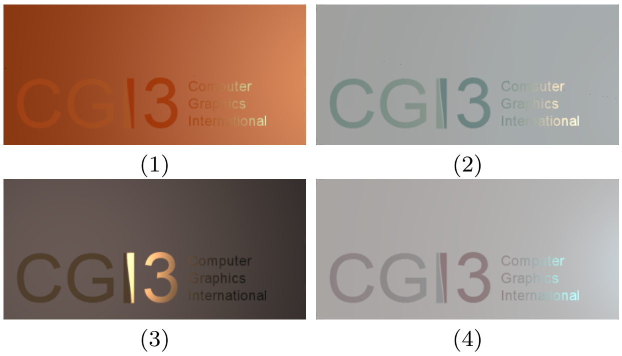

Fig. 3 CGI2013 logo rendered using different materials. Base materials: (1)$AlGaAs7$, (2)Barium fluoride(BaF_2), (3)Iron silicide($FeSi_2$), (4)Silicon oxynitride(40% N)

bottom and a thin painted layer on top. We provide an efficient model for computing subsurface BRDF for such materials by solving PRTE using the Ambartsumian's integral equation for the semi-inifinte base material, and invariant imbedding method for the paint layer.

We have shown our model's ability to correctly render a wide range of materials and we have validated our model by comparing with solutions from a standard RTE solvers available in the literature. In the future, we would like to extend our model to take into account polarization and multiple finite-layered materials.

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References

1. Ambarzumian, V.: On the problem of the diffuse reflection of light. J. Phys. Acad. Sci. USSR **8**, 65–75 (1944)
2. Blinn, J.: Light reflection functions for simulation of clouds and dusty surfaces. ACM SIGGRAPH Computer Graphics **16**(3), 21–29 (1982)
3. Chandrasekhar, S.: Radiative transfer. Dover publications (1960)
4. Chen, K., Collin, C., Hakke-Patil, A., Pattanaik, S.: A practical model for computing the brdf of real world materials. In: Proceedings of the ACM SIGGRAPH Symposium on Interactive 3D Graphics and Games, I3D '13, pp. 180–180. ACM, New York, NY, USA (2013)
5. de Haan, J.F., Bosma, P.B., Hovenier, J.W.: The adding method for multiple scattering calculations of polarized light. Astronomy and Astrophysics **183**, 371–391 (1987)
6. Donner, C., Jensen, H.W.: A spectral bssrdf for shading human skin. In: Proceedings of the 17th Eurographics conference on Rendering Techniques, EGSR'06, pp. 409–417. Eurographics Association, Aire-la-Ville, Switzerland, Switzerland (2006)
7. Farrell, T., Patterson, M., Wilson, B., et al.: A diffusion theory model of spatially resolved, steady-state diffuse reflectance for the noninvasive determination of tissue optical properties in vivo. Med. Phys **19**(4), 879–888 (1992)
8. Hanrahan, P., Krueger, W.: Reflection from layered surfaces due to subsurface scattering. In: Proceedings of the 20th annual conference on Computer graphics and interactive techniques, SIGGRAPH '93, pp. 165–174. ACM, New York, NY, USA (1993)
9. Hansen, J.E., Travis, L.D.: Light scattering in planetary atmospheres. Space Science Reviews **16**, 527–610 (1974)
10. van de Hulst, H.: Light scattering by small particles. Dover publications (1981)
11. Kubelka, P.: New contributions to the optics of intensely light-scattering materials. part ii: Nonhomogeneous layers. J. Opt. Soc. Am. **44**(4), 330–334 (1954)
12. Mishchenko, M.I., Dlugach, J.M., Yanovitskij, E.G., Zakharova, N.T.: Bidirectional reflectance of flat, optically thick particulate layers: an efficient radiative transfer solution and applications to snow and soil surfaces. Journal of Quantitative Spectroscopy and Radiative Transfer **63**(2-6), 409 – 432 (1999)
13. Nicodemus, F.E., Richmond, J.C., Hsia, J.J., Ginsberg, I.W., Limperis, T.: Radiometry. chap. Geometrical considerations and nomenclature for reflectance, pp. 94–145. Jones and Bartlett Publishers, Inc., USA (1992)
14. Pharr, M., Hanrahan, P.: Monte carlo evaluation of nonlinear scattering equations for subsurface reflection. In: Proceedings of the 27th annual conference on Computer graphics and interactive techniques, SIGGRAPH '00, pp. 75–84. ACM Press/Addison-Wesley Publishing Co., New York, NY, USA (2000)
15. Sobolev, V.V.: Light Scattering in Planetary Atmospheres. Pergamon Press (1975)
16. Software Spectra Inc.: Sopra optical database. URL http://www.sspectra.com/sopra.html
17. Stamnes, K., Tsay, S.C., Wiscombe, W., Jayaweera, K.: Numerically stable algorithm for discrete-ordinate-method radiative transfer in multiple scattering and emitting layered media. Appl. Opt. **27**(12), 2502–2509 (1988)
18. Stamnes, K., Tsay, S.C., Wiscombe, W., Laszlo, I.: Disort, a general-purpose fortran program for discrete-ordinate-method radiative transfer in scattering and emitting layered media: documentation of methodology
19. Torrance, K.E., Sparrow, E.M.: Theory for off-specular reflection from roughened surfaces. J. Opt. Soc. Am. **57**(9), 1105–1112 (1967)